Informe de actividades

Nicolás Grijalva y Ortiz

Período: 1 septiembre de 2 002 al 31 de marzo de 2 003.

Proyecto:

Generación de energía eléctrica a partir del oleaje

Este proyecto se refiere a un invento del Sr. Miguel Bretón que sometió a CONACYT para su evaluación. CONACYT lo envió al CICESE para la hacerla ya que no cuenta con los medios. CICESE invitó al suscrito para que se hiciera un proyecto de evaluación y fuera el responsable científico. Hasta la fecha se han realizado algunos modelos numéricos y se construyó dos modelos físicos de operación.

Se realizó un prototipo físico. Consiste en algunos componentes realizados en plásticos y de una turbina Pelton en metal. Este prototipo se enviará al CICESE.

A continuación se presenta la investigación matemática y numérica que se escribió en forma de artículo. Este artículo aún debe de ser editado.

An electric power generator worked by the surf

N, Grijalva. CICESE.

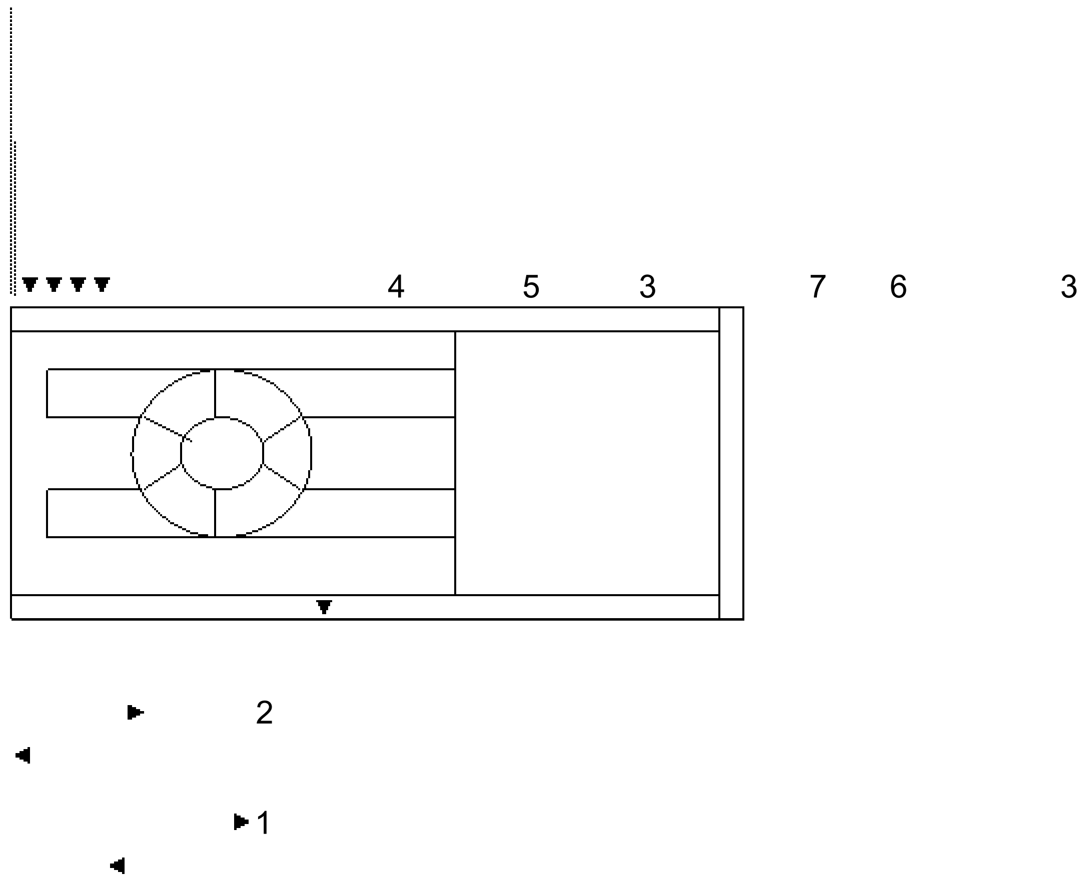
February, 2003

Summary

Last year a new device to capture the energy from the sea surf and to transform it into electric power was presented. This device is built in a breakwater or in a geological structure. It is required that the wave arrives to the place before breaking, with this it is achieved the water kinetic energy can be captured and transformed into electric energy.

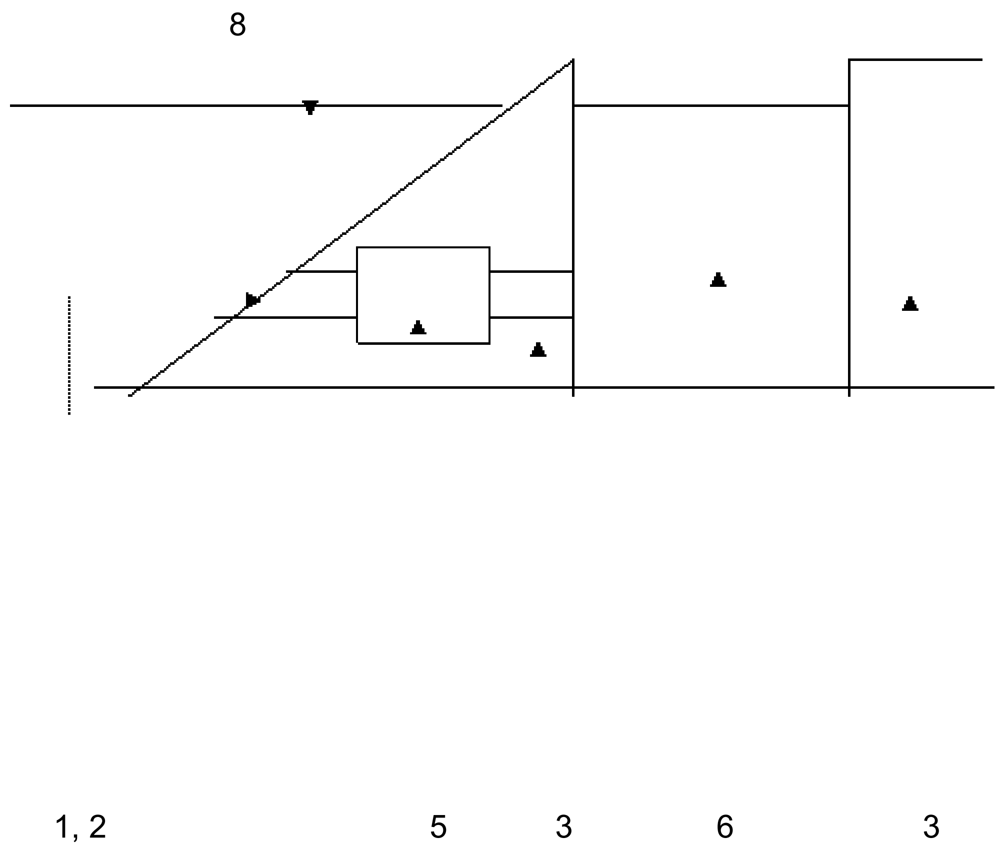
As shown in the diagram in Figure (1), it is possible to place a Pelton Turbine that rotates when the water enters the device, and also when it leaves it. The purpose of this paper is to build up a mathematical model of this device to be solved analytically and then build another to be solved numerically. We want to describe the motion of the water inside the structure and obtain a first opinion on the available energy. The hydrodynamic equations are used and an analytic

solution is attempted. In the second part a model of numeric simulation is built up. Interesting results are obtained.



Plant

1 Inlet
2 Outlet
3 Structure
4 Inlet check
5 Turbines
6 Resonance Chamber
7 Outlet Check
8 Sea levels



Cross Section

Figure 1. Diagram of the Device

1 THE THEORETICAL MODEL.

1 The Formula Model

The water movement in a region is described by the hydrodynamic partial differential equations, the boundary conditions and the initial conditions. In this case the Navier - Stokes and the continuity equations are used (Grijalva, N 1972). It is supposed that the non-linear terms are small and that, in first approach, the effects of viscosity are neglected:

$$\frac{\partial u}{\partial t} = -g\,\frac{\partial h}{\partial x} \qquad (1\text{ a})$$

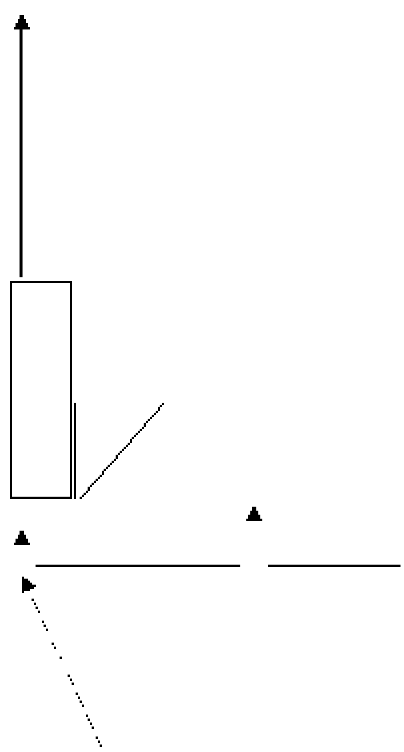
And the continuity equation:

$$uA_1 = vA_2 \qquad (1\text{ b})$$

Where:

x is the abscissa, parallel at the sea level
y is the ordinate, perpendicular to the sea level
u is the velocity component along the abscissa
v is the velocity component along the ordinate.
hw is the sea level measured in the open sea,
he is the water elevation inside the resonance chamber
g is the earth gravitation acceleration
A_1 and A_2 they are the areas of the pipe and of the Resonance Chamber cross sections

The second equation is the continuity one, that expresses that the amount that goes trough the pipe is the same that goes by the Resonance Chamber. (2 Giancolli, 1978)



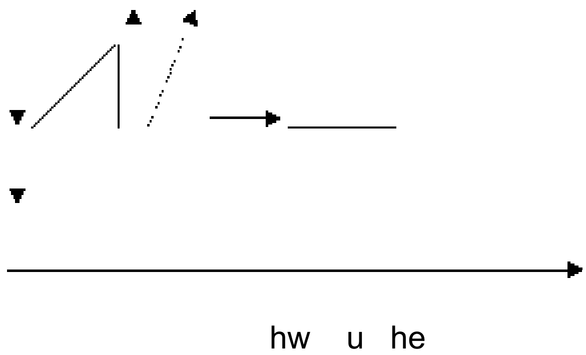


Figure 2. Diagram of the water velocities and the water elevations in the open sea and in the camera.

The vertical velocity is the derivative, with regard to the time, of the water level

$$v = \frac{\partial h}{\partial t}$$

$$u = \frac{A_2}{A_1}\frac{\partial h}{\partial t} = c^2(\partial h / \partial t)$$

It is substituted in (1 b):

The system (1) is expressed now as:

$$\frac{\partial u}{\partial t} + g\frac{\partial h}{\partial x} = 0$$
$$0 + c^2\frac{\partial h}{\partial t} = u \qquad (2)$$

Whose matrix operator is:

$$L_{i,j} = \begin{pmatrix} \frac{\partial}{\partial t} & g\frac{\partial}{\partial x} \\ 0 & c^2\frac{\partial}{\partial t} \end{pmatrix}$$

its characteristic form is:

$$C(Y) = \begin{vmatrix} Y_1 & gY_2 \\ 0 & c^2 Y_1 \end{vmatrix} = 0$$

Whose roots are $Y_1 = 0$ and $Y_2 = 0$.

Both are real, and therefore the equations are hyperbolic (Courant und Hilbert, 1937).

To outline the problem initial and boundary values must be prescribed. They are:

The initial one is that the water at the beginning is at rest:

$$u(0,x) = 0$$

and $$h(0,x) = 0$$

The boundary ones are:

The water flows parallel to the walls of the structure.

In the open frontier, the one in the ocean, the values of h are known:

$$hw = hw(t)$$

1 B ANALYTIC SOLUTION:

The solution is found by substituting the value of the water velocity that was found before in the first equation, after this is done we obtain:

$$c^2 \frac{\partial^2}{\partial t^2} h + g \frac{\partial}{\partial x} h = 0$$

and therefore:

$$c^2 \frac{\partial^2}{\partial t^2} h = -g \frac{\partial}{\partial x} h = -k^2$$

Where k is the separation coefficient (Ritger and Rose 2 000) Its value is: $k^2 = 0.0981$, considering a difference of 0.1 m between the sea level outside and inside the Resonance Chamber.

This it is a partial differential equation with two unknowns and two variables. The solution is the product of two functions of a single variable:

$$h(t,x) = T(t) \cdot X(x)$$

$$\frac{\partial^2}{\partial t^2} h = X(x) \cdot \frac{d^2}{dt^2} T$$

$$\frac{\partial}{\partial x} h = T \cdot \frac{d}{dx} X$$

This is substituted in (3) and the variables are separated:

$$\frac{d^2}{dt^2} T + \left(\frac{k}{c}\right)^2 T = 0 \quad \text{and}$$

$$\frac{d}{dt} X - \frac{k^2}{g} X = 0$$

Their immediate solutions are.

$$T = c_1 e^{i\omega t} \qquad \text{and,}$$

$$X = hwe^{-k^2 x / g}$$

The solution takes the form:

$$h(t, x) = c_1 hw \cdot e^{i\omega t - k^2 x / g}$$

The Resonance Chamber and the pipes are designed in such a way that $A_1 = 1$ and $A_2 = 4$

We obtain:

$$c = 2, \qquad k^2 = 0.981,\ g = 9.81,\ \omega = (k / c) = 0.4952$$

$T = 12.68$ is the Natural Oscillation Period.

The value of k was chosen proposing a level difference between the sea and the resonance chamber of 0.1 m.

The solution takes the form:

$$h(t, x) = hw \cdot e^{i0.4952t - 0.01x)}$$

,

(The units are in SI)

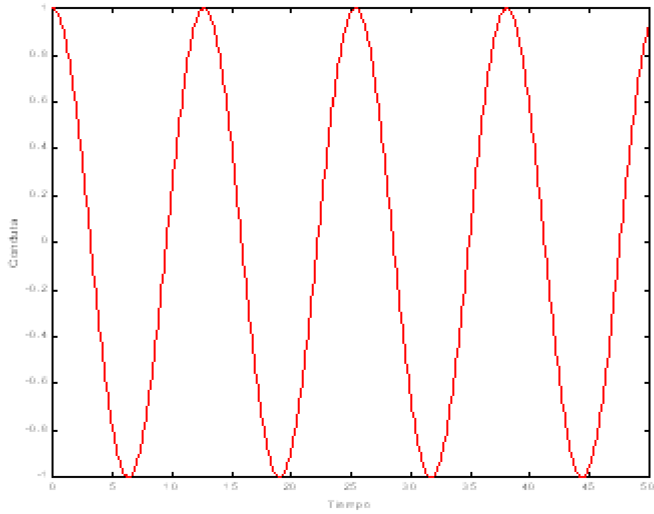


Figure 3. Graphic of $h(t) = e^{-i\omega t}$

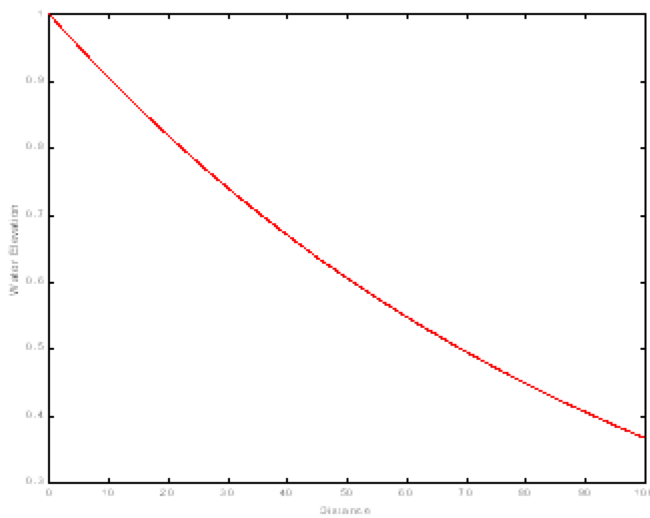


Figure 4. Graphic of $h(x) = e^{-0.1x}$

The first one presents an oscillation and the second a curve that falls logarithmically. But it exists the possibility that this device enters in resonance. The case occurs when both frequencies approach.

Let $hw = h_0 e^{-i\omega_0 t}$, then the solution is given by: $h(t,x) = h_0 \cdot e^{-i\omega_0 t} \cdot e^{-(i0.4952t+0.01x)}$, after a short calculation we conclude that:

$$h(t,x) = 2sen(\omega_0 - 12.68)t.sen(\omega_0 + 12.68)t / 4(\omega_0 - 12.68)$$

If the period chosen for the incoming wave is similar to the natural circular period then resonance will occur. This concept is useful for device design.

2 Period of observed waves

The problem of the wave prediction is a well-known and even not a solved one to whole satisfaction. It about to predict the wave measurements when one knows the intensity of the wind that blows on the surface. They are many the factors that intervene in this process. We are interested mostly in the period as we consider that the amplitude is in the order of 1 m. There are extraordinary cases in which the amplitude reach some two or thee meters but we do not take them in account because they rarely occur.

The incident wave is considered sinusoidal. We take into account the form that gives Netel (1995).

.

For this purpose of this work we take different authors' data as presented by Komar (1976).

Pierson, Neumann and James, 1955

	min	half	max
Hw	0.43m	31.4m	10.5m
T	4 s	22 s	10.22 s

Snodgrass: et al (1966)

T	10 s	30 s

Ocampo Torres et al 2001:

T	7.12 s

In this work we take values that are among these limits.

2 Numerical Simulation:

For this purpose we use the equations that were written up before:

$$\frac{\partial u}{\partial t} + g\frac{\partial h}{\partial x} = 0$$

$$u - g\frac{\partial h}{\partial t} = 0$$

Those derivatives with respect to the time are substituted by forward difference quotients, and those derived with regard to x are substituted by central differences ones:

$$\frac{\partial u}{\partial t} = \frac{un-u}{\Delta t};\ \frac{\partial h}{\partial t} = \frac{hn-h}{\Delta t}$$

$$\frac{\partial h}{\partial x} = \frac{he-hw}{\Delta x}.$$

We obtain:

$un = u - (g/k^2)(he-hw)\Delta t/\Delta x,$ and

$hn = h + (1/k^2)u \cdot \Delta t$

Which are the work equations. It is necessary to consider the initial conditions and to the boundary ones as they were presented before.

The condition in the ocean is:.

$hw(t) = h_1 \cos(\omega t) = h_1 \operatorname{Re}(e^{-i\omega t})$

The program in MatLab is:

```
%Este programa es para calcular la energía del aparato de Sr. Bretón.

%Valores iniciales

u=0;
he=0;
hw=0;
h=0.5;
g=9.81;
A1=1; A2=4;
dt=1; dx=8; ddt=10;
ea=0;epa=0;
itt=0;
```

```
ta=zeros(1,200);

%corre programa
for t=1:1:500
   z=t;
   itt=itt+1;
   if t<=100
      hw=h*(t/100)*cos(2*pi*t/10);
   else
    hw=h*cos(2*pi*(t-100)/10);
   end
un=u-g*(he-hw)*dt/dx-R*u;
u=un;
u3=(A1/A2)*u;
e=A1*(u.^2)/2;
hen=he+u*(A1/A2)*dt;
he=hen;
tt=t*dt;
ea=ea+e;
ta(1,z)=t;
ua(1,z)=u;
hea(1,z)=he;
hwa(1,z)=hw;
end
ea/3600
%ta
%u
plot(ta,hea,'r',ta,hwa,'b')
```

(MatLab Student Edition, 1996).

This program compute the water elevation in the Resonance Chamber and the velocity inside the pipe. It stores the data for later use. It also calculates the water kinetic energy of the in the pipe. Finally, it presents the results in graph form.

Results:

The water elevations are presented calculated for one hour. In this case the conditions were chosen:

$u = 0,$

$he = 0,$

$hw = 0$

For the first model the following values were considered:

$h = 1$

$hw = h\cos(2\pi t/10)\ ,$

$\Delta x = 10,\ \Delta t = 1,\ \ A_1 = 1,\ \ A_2 = 4$

At the beginning the condition was given that the value of it grew of zero up to 0.5 in an interval of time of 10 sec. The calculation is continued during 50 seconds more. The blue curve is the wave elevation in the open ocean and the red curve is the water elevation in the chamber

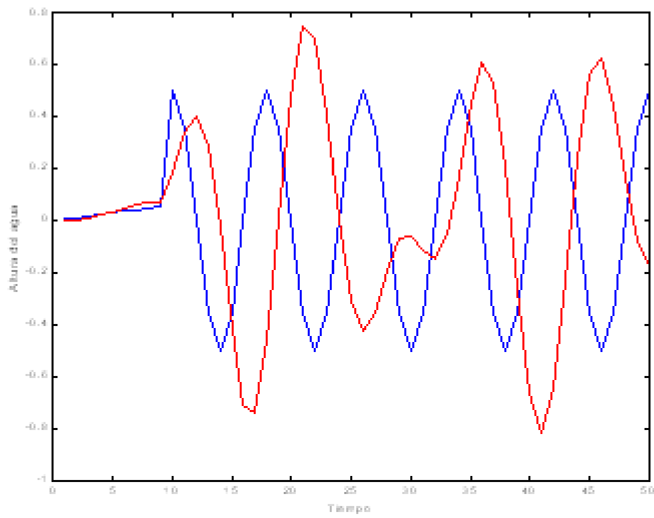


Figure 4. Results. Water Elevation inside (in red)
And outside (in blue) the Resonance Chamber.

This other graph shows the water elevations in the camera in red and the velocity in the tubes in blue

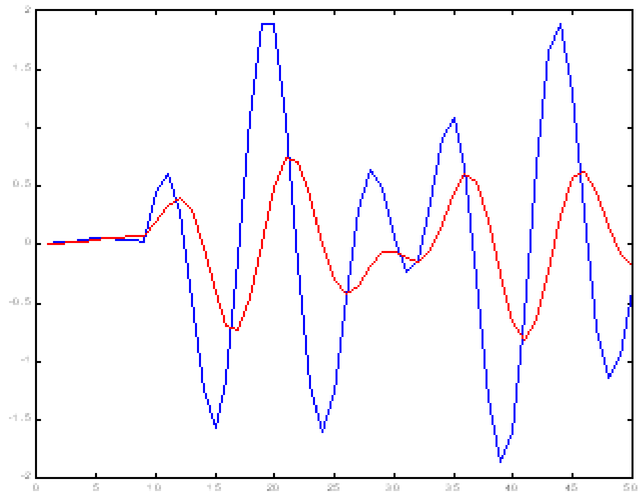


Figure 5. The velocity current in the pipe (red) and the water elevation (blue) in the Resonance Chamber

In this case the current and the water elevation were shown. Then several simulations were run to compute the energy that can be accumulate during one hour.

The velocity in the tube is shown in blue and the water elevation in the chamber in red. Notice you that the graph presents a great density of colors because the period is small compared with the computation time.

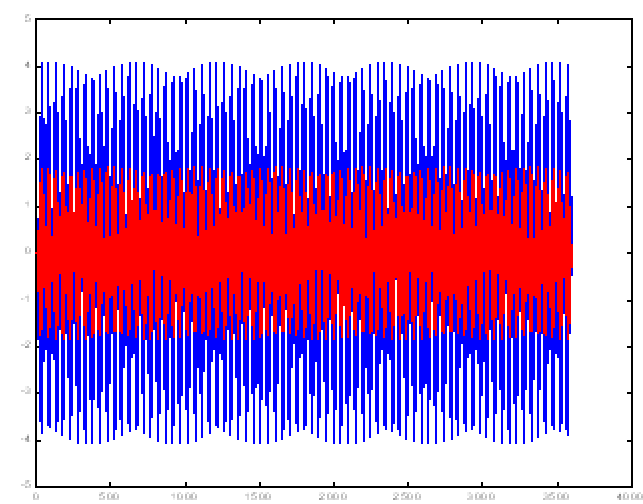


Figure 6. The current in the tube and the water elevation in the camera (during one hour)

The accumulated energy is 1.6835 MJ ($1MJ = 10^{6}\ jouls$)

Several experiments were done, their results are shown in Tables 1, 2, 3, 4, 5 and 6.

We show the amplitude of the incoming wave H.

Its period T, the distance of the two measurement locations ; Dist, the areas of both cross sections; A_1 and A_2 and the amount of energy accumulated; E.

H	T	Dist	A_1	A_2	E
0.5000	10.0000	10.0000	1.0000	4.0000	7.4877
0.5000	12.0000	10.0000	1.0000	4.0000	213.8509
0.5000	14.0000	10.0000	1.0000	4.0000	42.544
0.5000	16.0000	10.0000	1.0000	4.0000	9.7207
0.5000	18.0000	10.0000	1.0000	4.0000	4.9397
0.5000	20.0000	10.0000	1.0000	4.0000	3.294

Table 1

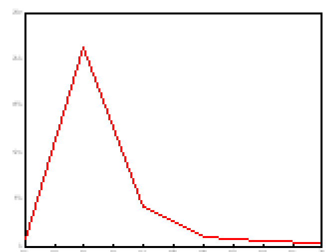


Figure 7

In figure 7 we present in the ordinate the energy accumulated during one hour. On the abscissa we represent the periods. Each mark means two unities beginning with 10 sec.

H	T	Dist	A_1	A_2	E
1.0000	10.0000	10.0000	1.0000	4.0000	29.9508
1.0000	12.0000	10.0000	1.0000	4.0000	855.4031.
1. 0000	14.0000	10.0000	1.0000	4.0000	170.1780
1.0000	16.0000	10.0000	1.0000	4.0000	38.8827
1.0000	18.0000	10.0000	1.0000	4.0000	19.7588
1.0000	20.0000	10.0000	1.0000	4.0000	13.1784

Table 2

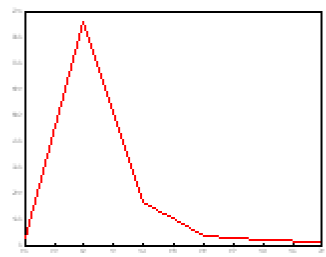


Figure 8

H	T	Dist	A_1	A_2	E
0.5000	10.0000	10.0000	1.0000	9.0000	1.5841
0.5000	12.0000	10.0000	1.0000	9.0000	3.4471
0.5000	14.0000	10.0000	1.0000	9.0000	8.7007
0.5000	16.0000	10.0000	1.0000	9.0000	30.8785
0.5000	18.0000	10.0000	1.0000	9.0000	371.7720
0.5000	22.0000	10.0000	1.0000	9.0000	51.3818

Table 3

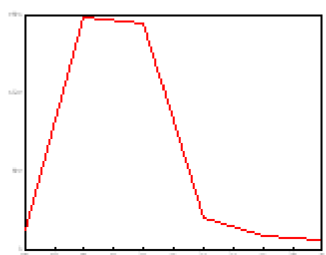


Figure 9

H	T	Dist	A_1	A_2	E
1.0000	16.0000	10.0000	1.0000	9.0000	123.5139
1.0000	18	10.000	1.000	9.000	1487.1
1.0000	20.000	0.0100	0.0010	0.0090	1438.3
1.0000	22.0000	10.0000	1.0000	9.0000	205.5270
1.0000	24.0000	10.0000	1.0000	9.0000	89.9357
1.0000	26.0000	10.0000	1.0000	9.0000	54.2071

Table 4

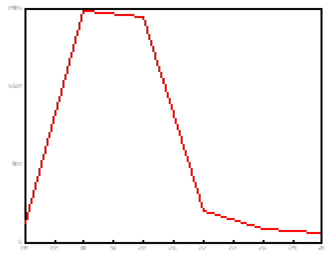


Figure 10

H	T	Dist	A_1	A_2	E
0.5000	10.0000	10.0000	1.0000	16.0000	1.0708
0.5000	20.0000	10.0000	1.0000	16.0000	26.3550
0.5000	22.0000	10.0000	1.0000	16.0000	79.7076
0.5000	24.0000	10.0000	1.0000	16.0000	613.3842
0.0005	0.0260	0.0100	0.0010	16.0	2626.9000
0.5000	28.0000	10.0000	1.0000	16.0000	193.5288
0.5000	30.0000	10.0000	1.0000	16.0000	71.8677

Table 5

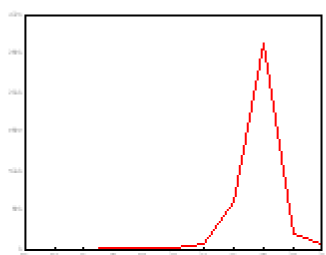


Figure 11

H	T	Dist	A_1	A_2	E
0.5000	20.0000	10.0000	1.0000	25.0000	8.8896
0.5000	22.0000	10.0000	1.0000	25.0000	15.2991
0.5000	24.0000	10.0000	1.0000	25.0000	28.1427
0.5000	26.0000	10.0000	1.0000	25.0000	60.1173
0.5000	28.0000	10.0000	1.0000	25.0000	164.9266
0.5000	30.0000	10.0000	1.0000	25.0000	880.6730
0.5000	32.0000	10.0010	1.0001	25.0025	22396.
0.5000	34.0000	10.0000	1.0000	25.0000	598.3836

Table 6

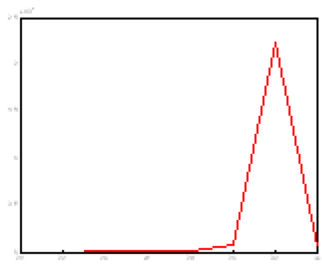


Figure 12

0.5000	34.0000	10.0000	1.0000	25.0000	598.3836
0.5000	34.0000	15.0000	1.0000	25.0000	92.7537
0.5000	34.0000	20.0000	1.0000	25.0000	14.0353
0.5000	34.0000	25.0000	1.0000	25.0000	5.1683
0.5000	34.0000	30.0000	1.0000	25.0000	2.5979
0.5000	34.0000	35.0000	1.0000	25.0000	1.5500

Table 7

In table 7. We write the energy and the rest of the parameters. Note the variation introduced on the distance along the abscissa.

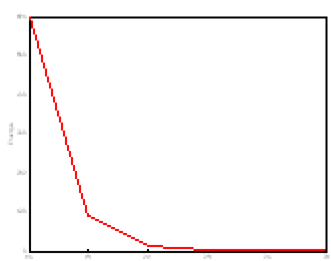


Figure 13

In Figure 13 we plot the energy against the distance. When we approach the left side the energy is larger and then decreases slowly toward the right side as in Figure 4.

Conclusion

The results shown here have to be taken with care. The models do not take in account the non linear terms which include the advection and the friction. The energy computed is the kinetic energy in the water and there always exist dissipation when it is converted to electrical electricity. It is also to be taken in consideration that the energy in the water is not constant. When the water acquires a maximum velocity is when the energy is large. Then the motion slows down to zero. After this the velocity increases to another maximum and finally comes down to rest.

We suggested using a Pelton Wheel which ca be the ideal exchanger for the acquisition of electric energy. Then again it can be substituted by another device more suitable for this purpose.

Nevertheless this model suggests that the invention has possibilities to work properly. About the wave it is possible to say that before the wave breaks the motion can be approximated by a sinusoidal motion. The amplitudes are not constant and neither are the frequencies, but most will have the proper values in such a way that they will produce energy.

This paper suggests that research should be continued both on theoretical and experimental work. When this is achieved, and proved positively, then the development of a commercial prototype can be done

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